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AN ANALYTICAL SOLUTION OF FLUID–STRUCTURE COUPLING OSCILLATION IN ONE-DIMENSIONAL IDEAL CONDITION UNDER SMALL DISTURBANCE

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1. INTRODUCTION

Fluid–structure coupling vibration widely exists in nature, e.g., the mutual actions between the vibrant elastic wings of a plane and the surrounding unsteady air, a flickering bird's wings and the surrounding air, a large-scale flexible architectural structure and the surrounding air, etc. The interaction of elastic structure and oscillating flow field is frequently encountered in engineering. Liu *et al.* [1] proposed a hybrid numerical method for studying elastic stress waves in a composite laminate subjected to a plane shock wave. Xi *et al.* [2] and Liu *et al.* [3] proposed a strip element method to analyze the wave scattering by a crack in a fluid-filled composite cylindrical shell and in an immersed composite laminate. Liu *et al.* [4] proposed an analytical method to analyze elastodynamic response of an immersed composite laminate subjected to a Gaussian beam pressure. Gorman *et al.* [5] have studied the vibration of a flexible pipe conveying viscous pulsating fluid flow. The unsteady aerodynamic forces acting on the elastic airfoil or cascade and the dynamic responses of airfoil or blades were evaluated using a numerical method by Kim and Lee [6], Hsu and Chen [7], Taylor and Vezza [8] and Takahara and Masuzawa [9].

There are some analytical solutions to the dynamic problems of some simple or simplified elastic structure and some analytical solutions to the dynamic problems of some simplified oscillating flow field [10]. However, the coupling dynamic oscillation of fluid–structure is much more complex. So far, even for the simplest model of fluid–structure, the analytical solution cannot be found in references.

In this article, an analytical solution will be given for a simplified model of fluid–structure system. The analytical solution is helpful to understand some essential, inherent and important characteristics of the fluid–structure system, and can also be used to verify the results of numerical simulation to this kind of problems.

2. FLUID–STRUCTURE COUPLING RELATIONSHIP

When steady flow field is still, the governing equations, given by Chen [10], can describe the dynamic movement for a one-dimensional oscillating ideal gas under small disturbance as follows:

$$\begin{aligned} &\frac{\partial \hat{\rho}}{\partial t} + \frac{\partial(\tilde{\rho}\hat{u})}{\partial x} = 0, \\ &\frac{\partial \hat{u}}{\partial t} + \frac{\gamma \tilde{p}}{\tilde{\rho}^2}\frac{\partial \hat{\rho}}{\partial x} = 0, \end{aligned} \tag{1}$$

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where $\hat{u}$, $\hat{\rho}$, $\hat{p}$ are the oscillating velocity, oscillating density and, oscillating pressure respectively. $\tilde{\rho}$ is the steady density, $\tilde{p}$ steady pressure and γ the specific heat ratio. Huang [11] gives the following analytical solutions to equation (1):

$$\begin{aligned}\hat{u} &= \sum_i (C_i \cos r_i x + D_i \sin r_i x)\cos \omega_i t,\\ \hat{\rho} &= \sqrt{\frac{\tilde{\rho}^3}{\gamma \tilde{p}}}\sum_i (C_i \sin r_i x - D_i \cos r_i x)\sin \omega_i t,\\ \hat{p} &= \sqrt{\gamma \tilde{p}\tilde{\rho}}\sum_i (C_i \sin r_i x - D_i \cos r_i x)\sin \omega_i t.\end{aligned} \tag{2}$$

where ω_i is the frequency of the ith oscillating component, C_i, D_i are constants of the ith oscillating component, which are fixed by the boundary conditions of the beginning and the end of the one-dimensional flow field. r_i is used to present $\omega_i\sqrt{\tilde{\rho}/\gamma\tilde{p}}$, that is, $r_i = \omega_i\sqrt{\tilde{\rho}/\gamma\tilde{p}}$.

In Figure 1, E and F are ideal sliding boards with one unit area, mass m respectively. G, a simplified model of one-dimensional flexible structure, is made up of board F and the connected spring. The dynamic response of board F can be described by the following equation:

$$m\ddot{x}_F + k_1 x_F = \hat{p}, \tag{3}$$

where k_1 is the rigidity of structure G. The balance position of F is x_0, and x_F is the relative displacement to x_0.

It is very clear that at the interface of one-dimensional ideal flow field and the sliding board E or F, the oscillating velocity and acceleration of flow field are equal to the vibrating velocity and acceleration of board E and F at their relevant frequencies respectively.

When board E is moving, its vibration can be regarded as the superposition of some simple harmonic vibrations; thus,

$$\begin{aligned}x_E &= \textstyle\sum_i A_i \sin \omega_i t,\\ u_E &= \textstyle\sum_i \omega_i t A_i \cos \omega_i t,\end{aligned} \tag{4}$$

where A_i is the amplitude of vibration of ith oscillating component, ω_i is the frequency of the ith oscillating component, and x_E is the displacement of board E.

Substituting equation (4) into equations (2), and omitting the second order small quantity,

$$C_i = A_i \omega_i. \tag{5}$$

From equation (2), the oscillation pressure at any position of the flow field can be written as

$$\hat{p} = \sqrt{\gamma \tilde{p}\tilde{\rho}}\sum_i (A_i\omega_i \sin r_i x - D_i \cos r_i x)\sin \omega_i t. \tag{6}$$

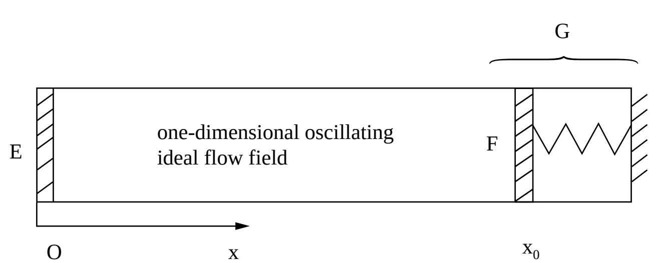

Figure 1. Sketch map of a simplified model for the fluid–structure system.

When the vibration of board E is small, the oscillating pressure on board F is obtained, neglecting the second order small quantity on board F.

$$\hat{p} = \sqrt{\gamma\tilde{p}\tilde{\rho}}\sum_i (A_i\omega_i \sin r_i x_0 - D_i \cos r_i x_0)\sin \omega_i t. \tag{7}$$

By integrating and differentiating the oscillating velocity $\hat{u}$ of equation (2), the displacement and acceleration of the flow field on board F are shown as

$$x_F = \sum_i \frac{(A_i\omega_i \cos r_i x_0 + D_i \sin r_i x_0)}{\omega_i}\sin \omega_i t,$$

$$\ddot{x}_F = -\sum_i \omega_i (A_i\omega_i \cos r_i x_0 + D_i \sin r_i x_0)\sin \omega_i t. \tag{8}$$

Substituting equations (7) and (8) into equation (3),

$$D_i = \frac{A_i\omega_i\left(k_1 \cos r_i x_0 - m\omega_i^2 \cos r_i x_0 - \omega_i\sqrt{\gamma\tilde{p}\tilde{\rho}}\sin r_i x_0\right)}{m\omega_i^2 \sin r_i x_0 - \omega_i\sqrt{\gamma\tilde{p}\tilde{\rho}}\cos r_i x_0 - k_1 \sin r_i x_0}. \tag{9}$$

So far, the vibrating velocity, acceleration and pressure of any position in flow field according to equation (2) can be known, and also the dynamic response of board F from equation (3). As shown in equation (9), the rigidity and mass of the simplified structure G influence the oscillating flow field. Also, any change in these parameters of geometry (x_0), unsteady density and pressure of fluid can bring exciting force to the simplified structure G, and result in a change of its dynamic characteristics. Hence, it is a coupling system.

3. ANALYSIS AND DISCUSSION

(1) For $\cos r_i x_0 = 0$:
From equation (9),

$$D_i = \frac{-\omega_i^2\sqrt{\gamma\tilde{p}\tilde{\rho}}A_i}{m\omega_i^2 - k_1}.$$

It is obvious that if $m\omega_i^2 - k_1 = 0$, then $D_i \to \infty$, the inherent frequency of the fluid–structure coupling system is just the same as that of the structure G ($\omega_F = \sqrt{k_1/m}$), and not relative to the parameters of oscillating flow field.

(2) For $\sin r_i x_0 = 0$:
From equation (9),

$$D_i = \frac{A_i\left(k_1 - m\omega_i^2\right)}{-\sqrt{\gamma\tilde{p}\tilde{\rho}}}.$$

This is an interesting result. When the disturbance frequency ω_i equals the inherent frequency of structure ω_F, $D_i = 0$. If the disturbance frequency ω_i is not equal to the inherent frequency of structure ω_F, D_i is a limited value, which is relative to the mass and rigidity of the simplified structure and parameters of steady flow field. In this case, the fluid–structure system does not have inherent frequency in the usual sense. There is no resonance in the system, no matter what frequency of exciting force is used to stimulate the fluid–structure system. This can be called a super-steady system. For the fluid–structure system, it is an important discovery and can be used to guide in the design of a stabile system. In practical engineering, $\sin r_i x_0$ should be as near zero as possible.

(3) For $\sin r_i x_0 \neq 0$ and $\cos r_i x_0 \neq 0$:

If the denominator of equation (9) is zero, $D_i \to \infty$, namely,

$$m\omega_i^2 \sin r_i x_0 - \omega_i \sqrt{\gamma \tilde{p} \tilde{\rho}} \cos r_i x_0 - k_1 \sin r_i x_0 = 0. \tag{10}$$

Equation (10) is non-linear. With the method of Newtonian iteration, the solutions $(\omega_i^{'})$ of ω_i can be obtained. If $\omega_i^{'} < 0$, it is obviously nonsense, since the frequency of disturbance cannot be negative. So for $\omega_i = \omega_i^{'} > 0$, $\omega_i^{'}$ is the inherent frequency of this system, $D_i \to \infty$. According to equations (2) and (3), the oscillation of flow and the vibration of structure become infinity, and the system is unstable. This frequency is not only relative to the mass and rigidity of the simplified structure, but also to the fluid parameters and geometric conditions. Thus, the system's inherent frequency cannot be simply regarded as only relative to inherent frequency of the simplified structure in this case.

4. CONCLUDING REMARKS

With mathematic deduction, an analytical solution to fluid–structure coupling oscillation in one-dimensional ideal condition under small disturbance is acquired. Some interesting and essential characteristics of fluid–structure coupling system oscillation are found. In general, for $\sin r_i x_0 \neq 0$ and $\cos r_i x_0 \neq 0$, the fluid–structure coupling system has a natural frequency that is not only related to the natural frequency of the solid elastic structure but also to the conditions of flow and geometric parameters. In some special conditions of $\cos r_i x_0 = 0$, the coupled natural frequency is only relative to that of the solid elastic structure. For $\sin r_i x_0 = 0$, there is no natural frequency in general for the system in this case, namely resonance does not occur. The above analytical solution and discovery are not only important to the theory of fluid–structure coupling oscillation, but can also be used as a standard solution to verify numerical solution for a number of actual fluid–structure coupling systems.

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REFERENCES

1. G. R. Liu, K. Y Lam and E. S. Chan 1996 *Shock and Vibration* **3**, 419–433. Stress waves in composite laminates excited by underwater shock wave.
2. Z. C. Xi, G. R. Liu, K. Y. Lam and H. M. Shang 2000 *Composite Science and Technology* **60**, 1985–1996. A strip element method for analyzing wave scattering by a crack in a fluid-filled composite cylindrical shell.
3. G. R. Liu, Z. C. Xi, K. Y. Lam and H. M. Shang 1999 *Journal of Applied Mechanics* **66**, 898–903. A strip element method for analyzing wave scattering by a crack in an immersed composite laminate.
4. G. R. Liu, X. J. Wang and Z. C. Xi 2000 *Journal of Sound and Vibration* **233**, 813–833. Elastodynamic responses of an immersed composite laminate subjected to a Gaussian beam pressure.
5. D. G. Gorman, J. M. Reese and Y. L. Zhang 2000 *Journal of Sound and Vibration* **230**, 379–392. Vibration of a flexible pipe conveying viscous pulsating fluid flow.
6. D.-H. Kim and I. Lee 2000 *Journal of Sound and Vibration* **234**, 859–880. Transonic and low-supersonic aeroelastic analysis of a two-degree-of-freedom airfoil with a freeplay non-linearity.
7. W. I. Hsu and S. H. Chen 2000 *Transactions of the Aeronautical and Astronautical Society of the Republic of China* **32**, 97–104. Unsteady flow computation of a vibrating cascade.

8. I. J. TAYLOR and M. VEZZA 1999 *Wind and Structures, an International Journal* **2**, 285–303. Computation of unsteady wind loading on bluff bodies using a discrete vortex method.
9. S. TAKAHARA and C. MASUZAWA 1998 *Memoirs of the Faculty of Engineering, Kyushu University* Vol. **58**(4), 199–216. Unsteady numerical analysis of stall flutter in turbine cascade.
10. Z. CHEN, Z. JIANG and X. SUN 1998 *Oscillating Fluid Dynamics*. Beijing: Hydrodynamic and Electric Publishing Company of China.
11. D. HUANG 1999 *Journal of Engineering for Thermal Energy & Power* **14**, 403–405. Analytical solution of an ideal gas one-dimensional non-viscous compressible oscillation flow.